FISH 502 Assignment #6
DUE: March 6, 2009
Equilibrium Analysis of Age-Structured Population Models to Estimate Reference Points

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There are two very important issues to consider in maximizing yield from natural fish populations, 1) the age at which fish are first harvested, and 2) the fishing mortality rate that is necessary to maximize long-term yield. Simple yield per recruit models consider growth and mortality but lack the compensation effects due to relative improvements in juvenile survival rates as stocks decline in abundance. Lou Botsford developed a rather eloquent method to integrate per-recruit methods with stock-recruitment effects (see Box 3.1 on page 56 in Walters and Martell, 2004, for a more detailed description).

Botsfords' method is based on setting up a series of per-recruit "incidence functions", for example the fecundity per-recruit or biomass per recruit. These incidence functions are usually denoted by the symbol ϕ and the subscript represents the type of incidence function (e.g., ϕ_e = eggs per recruit, ϕ_B = biomass per recruit, and for example, the total biomass in the stock is given by total recruits times biomass per recruit $B = R\phi_B$). Note that capital subscripts denote unfished conditions and lower case denotes fished conditions. The incidence functions are simply a weighted sum, where the weights are based on the survivorship (ι_a) to age a.

For your assignment, construct an equilibrium age-structured model using parameter values (Θ) defined in Table 1 and the model equations listed in Table 2. Note that the model in Table 2 is intended to represent the order of operations and Data, Parameters and Procedures are clearly separated.

Before going on to answer the questions you can verify that your model is working correctly by ensuring that R_o and R_e are equal when $f_e = 0$. You should also provide plots of equilibrium yield (Y_e), biomass (B_e), recruits (R_e) and spawn potential ration (SPR) versus equilibrium fishing mortality rates (f_e).

Questions

1. Using the parameters defined in Table 1, what is the optimal fishing mortality rate (F_{msy}) to maximize the long-term equilibrium yield? What is the maximum fishing mortality (F_{max}) rate that eventually leads to extinction. When fishing at F_{msy}, how much is the biomass and recruitment depleted relative to its unfished state?

Table 1: Parameter descriptions and assumed values.

$R_o = 100$	Unfished age-1 recruits
$\kappa = 12$	Recruitment compensation parameter
$M = 0.23$	Instantaneous natural mortality rate
$W_\infty = 1$	Asymptotic body weight (kg)
$k = 0.153$	Growth coefficient
$\dot{a} = 4.5$	Age at 50% maturity
$\sigma_{\dot{a}} = 1.5$	SD in age-at-maturity
$\hat{a} = 5.0$	Age at 50% vulnerability
$\sigma_{\hat{a}} = 1.25$	SD in age-at-vulnerability
$f_e = \{0, \ldots, f_{max}\}$	Equilibrium fishing rate
Derived variables	
w_a	mean weight-at-age
m_a	proportion mature-at-age
v_a	mean vulnerability-at-age
ι_a	survivorship to age a
ϕ_E	eggs per recruit
ϕ_b	biomass per recruit
ϕ_v	vulnerable biomass per recruit
R_e	Equilibrium recruitment
Y_e	Equilibrium yield
B_e	Equilibrium biomass
SPR	Spwan potential ratio

2. How sensitive is the SPR calculation to alternative values of the recruitment compensation parameter (κ), or $\dfrac{\partial \phi_e}{\partial \kappa}$?

3. What can you conclude about the effects of natural mortality rate on equilibrium yield (i.e., is this an important parameter for fisheries management, if so why?).

4. What can you conclude about the effects of the growth coefficient (k) on the estimate of the optimal fishing mortality rate?

5. Explain how the recruitment compensation parameter is related to population resilience (i.e., how well can the population recover from a severely over-fished state)?

6. What could you recommend to fisheries managers to improve the total yield to the fishery? What risks, if any, would this impose on the

Table 2: Equilibrium age-structured model.

DATA SECTION

A=15, a is the index for age

PARAMETER SECTION

$$\Theta = (R_o, \kappa, M; W_\infty, k, \dot{a}, \sigma_{\dot{a}}; \hat{a}, \sigma_{\hat{a}}, f_e)$$

PROCEDURE SECTION

$$w_a = W_\infty (1 - \exp(-ka))^3$$

$$m_a = \{1 + \exp(-(a - \dot{a})/\sigma_{\dot{a}})\}^{-1}$$

$$v_a = \{1 + \exp(-(a - \hat{a})/\sigma_{\hat{a}})\}^{-1}$$

$$\iota_a = \begin{cases} \exp(-M)^{a-1}, & 1 \leq a < A \\ \exp(-M)^{a-1}/(1 - \exp(-M)) & a = A \end{cases}$$

$$\breve{\iota}_a = \begin{cases} 1, & a = 1 \\ \breve{\iota}_{a-1} \exp(-M - f_e v_{a-1}), & 1 < a \leq A \\ \breve{\iota}_a/(1 - \exp(-M - f_e v_a)) & a = A \end{cases}$$

$$\phi_E = \sum_a \iota_a w_a m_a, \quad \phi_e = \sum_a \breve{\iota}_a w_a m_a$$

$$\phi_B = \sum_a \iota_a w_a, \quad \phi_b = \sum_a \breve{\iota}_a w_a$$

$$\phi_v = \sum_a \frac{\breve{\iota}_a w_a v_a (1 - \exp(-M - f_e v_a))}{M + f_e v_a}$$

$$R_e = R_o \frac{\kappa - \phi_E/\phi_e}{\kappa - 1}, \quad \kappa > 1.0$$

$$Y_e = f_e R_e \phi_v$$

$$B_e = R_e \phi_b$$

$$\text{SPR} = \phi_e/\phi_E$$

stock or our capacity to implement safe harvest strategies?

Bonus Question

Certainty in fisheries stock assessment is more or less related to certainty in parameter estimates. How would you go about estimating the uncertainty in F_{msy} using the model described in Table 2, given uncertainty in estimates of natural mortality rate M? For example, if the marginal posterior distribution for M is roughly approximated by a normal distribution with a mean $\mu_M = 0.23$ and a standard deviation $\sigma_M = 0.023$, how would you go about calculating (numerically) the implied distribution for F_{msy} as shown in Figure 1.

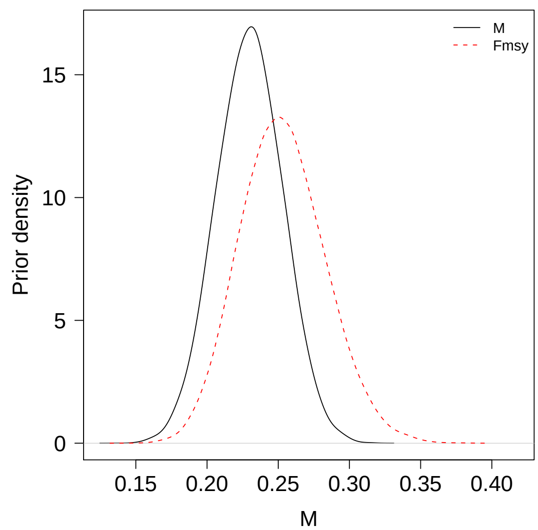

Figure 1: Prior density function for M (solid line) and the implied prior density function for F_{msy} (dashed line) based on the model in Table 2.

References

Walters, C. J. and Martell, S. J. D. 2004. *Fisheries Ecology and Management.* Princeton University Press, Princeton, NJ.